COMP0212: Modelling and Simulation - Coursework

Aditya Bishnoi
MEng Robotics and Artificial Intelligence
University College London
London
aditya.bishnoi.23@ucl.ac.uk

Rehan Agrawal
MEng Robotics and Artificial Intelligence
University College London
London
rehan.agrawal.23@ucl.ac.uk

1 Introduction

The purpose of this project is to evaluate how effective the basic Black-Scholes model is compared to modified models in simulating real option prices in the stock market. Specifically, we aim to investigate how randomness in volatility and interest rates affects the option premium price and how closely these simulated prices align with actual market data.

That sounds like a lot of jargon so let me break it down. Firstly, what are options? They are contracts created by other people, promises for those people who enter the contract. They promise to buy a stock of a certain company right now as you enter the promise (at the current stock price at that time) and sell all of that stock to you before the expiration date of the contract at a pre-agreed price (strike price), regardless of how the stock may have changed. You don't have to buy(execute the contract/option/promise), but if you ever do decide, then you must do it before the contract/option/promise expires. In order to jump into this contract, you must pay a fee (a premium) which can change based on the current stock market at that time. This is decided by the seller of the option. If the stock goes up and your premium (fee) plus strike price (pre-agreed price to buy) is below that value, then when you execute the contract and you will have expensive stock for a cheaper amount. If not, you can wait out the contract and never execute/buy and only lose the premium (fee) you paid to enter the contract.

The finance industry, worth trillions of dollars, has struggled to model and predict stock options, with the Black-Scholes equation being a key part of this effort. Many well-known figures and organizations have contributed to this field. Fischer Black and Myron Scholes developed the Black-Scholes model, which is widely used in finance. Benoît Mandelbrot studied the randomness and patterns in market prices. Companies like Goldman Sachs, Morgan Stanley, and Renaissance Technologies have invested heavily in creating better models. Physicists like Didier Sornette have applied their expertise to understand market crashes. Despite all these efforts, no perfect solution exists, making it an important and relevant area to study. The stock market is an excellent example of how theoretical models can be compared to real-world events.

We chose this project because the stock market is highly random and influenced by many factors, events, and complex interactions. This makes it a great topic for creating simulations and studying how different factors affect these models.

Objectives

- Assess the accuracy of the Black-Scholes model in predicting option prices.
- Investigate the impact of randomness in black-Scholes with variable volatility and interest rates on the option premium price.
- Create and analyze simulations of the option call price premium over time, up to the expiration date, using the following parameters:
  - Current stock price (from a data sample).
  - A fixed strike price for the option.
  - Time to maturity as a variable.
  - A constant risk-free interest rate.
  - A constant volatility (in the initial simulation).
- Simulate option premiums using the Black-Scholes PDE
- Simulate options premiums using the Black-Scholes PDE with randomness added as an external variable.
- Simulate option premiums using the Black-Scholes PDE, with and without randomness, but with variable volatility.
- Simulate option premiums using the Black-Scholes PDE, with and without randomness, but with variable interest rates.
- Compare simulations using statistics with varying assumptions to determine their accuracy and reliability.

2 Theoretical Background

2.1 System Description

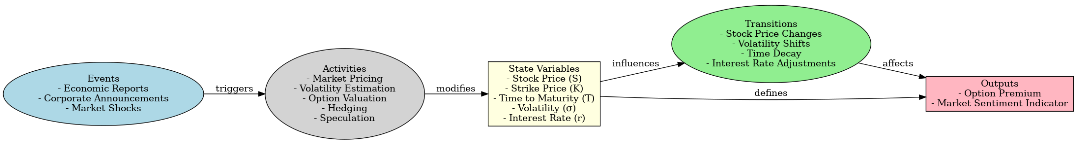

Figure 1: Diagram of the Stock Market Options System

The stock market options system focuses on pricing financial instruments called ”options,” which derive their value from underlying assets like stocks. The system is influenced by various events, activities, and measurable variables that work together dynamically. Events such as economic reports, corporate announcements, and sudden market shocks create changes in the market, often leading to shifts in stock prices and volatility.

Key activities within the system include market pricing, where continuous buying and selling determine the value of stocks and options, and volatility estimation, which measures price fluctuations to assess market uncertainty. Options are valued using models like Black-Scholes, which consider factors such as stock price, strike price, volatility, interest rates, and time to expiration. These models also account for activities like hedging, where traders manage risks, and speculation, where participants attempt to profit from market movements.

The system relies on several state variables that define its condition, including the current stock price, the strike price of the option, time remaining until the option expires, market volatility, and interest rates. These variables interact dynamically, influenced by transitions such as stock price changes, volatility shifts, or time decay. For example, as the expiration date approaches, an option’s time value decreases, which impacts its price unless offset by changes in volatility or stock price.

Feedback loops also play a significant role. For instance, increased volatility may drive up option premiums, affecting trader behavior and market liquidity. Similarly, rising stock prices may boost optimism, further driving prices higher, or trigger corrections during a sell-off. The primary output of this system is the option premium, which reflects the cost of buying or selling the option and serves as an indicator of market sentiment and perceived risk.

In summary, the stock market options system is a dynamic, interconnected network where external events, trading activities, and state variables constantly interact to determine option prices. This system provides insights into market behavior, uncertainty, and the complex interplay between participants and external forces.

2.2 Simulation Flowchart

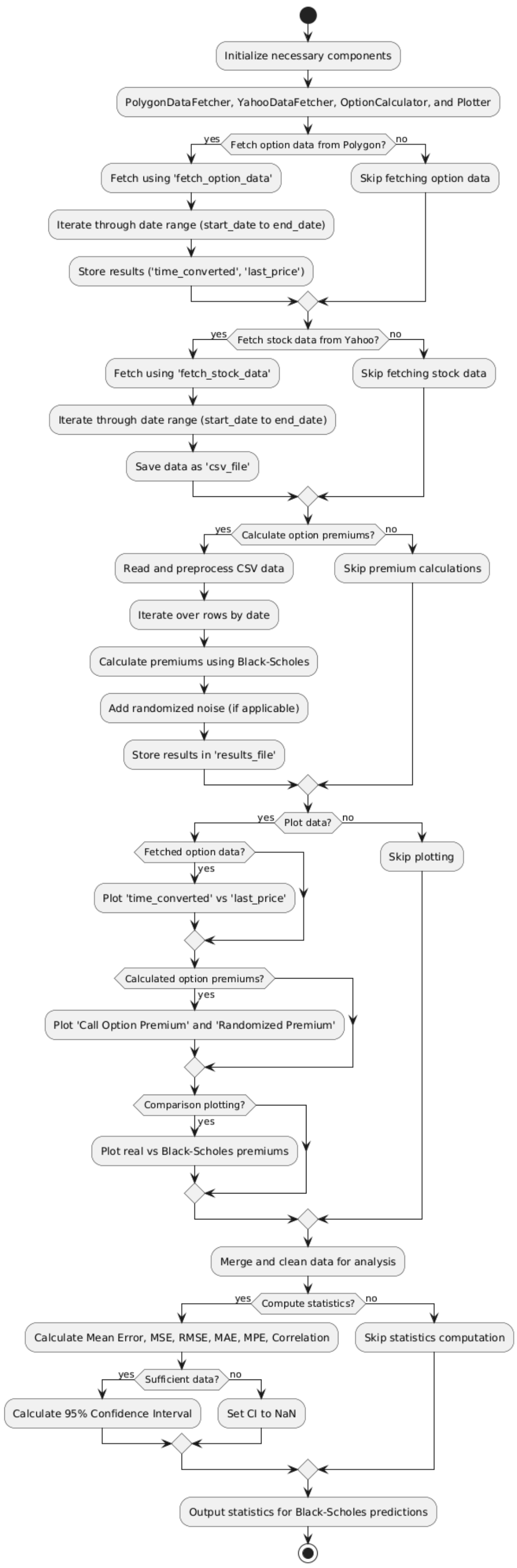

Figure 2: Simulation Flowchart

2.3 Mathematical Sections

2.3.1 1. Partial Differential Equations (PDEs)

The Black-Scholes equation is a partial differential equation (PDE) that describes how the value of an option depends on factors like stock price, volatility, time to maturity, and the risk-free interest rate. The PDE is given by:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$

Here:

- V: the value of the option.
- S: the current stock price.
- σ: the volatility of the stock, representing its price fluctuations.
- t: time to maturity (measured as the time remaining until the option expires).
- r: the risk-free interest rate, which is the theoretical return on a risk-free investment.

This equation is derived to ensure the option value $V(S,t)$ evolves in a way consistent with market dynamics. Below, we detail the derivation process, explain the assumptions and talk about financial modelling principles used.

—

2.3.2 2. Derivation of the Black-Scholes Model

Step 1: Assumptions of the Model The derivation begins by modeling the stock price as a stochastic process. The key assumption is that stock prices follow a *geometric Brownian motion*, described by:

$$dS = \mu S dt + \sigma S dW$$

where:

- μ: the expected rate of return of the stock.
- σ: the volatility of the stock.
- dW: a Wiener process representing random changes in the stock price.

This equation captures the behavior of stock prices by combining a deterministic drift ($\mu S dt$) - system's change that is determined by a fixed rule or equation - with a random component ($\sigma S dW$). This randomness reflects real-world uncertainties in price movements.

The risk-free interest rate r is assumed to be constant, and arbitrage-free market conditions are assumed (no risk-free profit opportunities exist).

—

Step 2: Applying Ito's Lemma To understand how the value of an option, $V(S,t)$, changes over time, we need to consider how it depends on both the stock price, S, and time, t. For this purpose, we use a tool from stochastic calculus called **Ito's Lemma**. This allows us to describe the evolution of a function of a stochastic process.

First, according to Ito's Lemma, the total change in $V(S,t)$ is given by:

$$dV = \frac{\partial V}{\partial t}dt + \frac{\partial V}{\partial S}dS + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}dS^2$$

This equation separates the total change in V into three components:

- $\frac{\partial V}{\partial t}dt$: The change in V due to the passage of time, t, while keeping S constant.
- $\frac{\partial V}{\partial S}dS$: The change in V due to a small change in S, the stock price.
- $\frac{1}{2}\frac{\partial^2 V}{\partial S^2}dS^2$: The change in V due to the second-order (curvature) effect of S (helps in capturing nonlinearities).

Next, we substitute the stochastic process for the stock price, S. The stock price follows the stochastic differential equation:

$$dS = \mu S dt + \sigma S dW,$$

where:

- $\mu S dt$: The drift term, representing the expected change in the stock price over a small time step. This is deterministic so it dependent on equations rather than random probability.
- $\sigma S dW$: The stochastic term, representing the random fluctuations in the stock price due to market volatility.

Additionally, the square of the stochastic term is given by:

$$dS^2 = (\sigma S dW)^2 = \sigma^2 S^2 dt,$$

because the variance of the Wiener process, dW^2, equals dt in stochastic calculus.

Substituting dS and dS^2 into the expression for dV, we get:

$$dV = \frac{\partial V}{\partial t} dt + \frac{\partial V}{\partial S}(\mu S dt + \sigma S dW) + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}\sigma^2 S^2 dt.$$

Expanding each term, we have:

$$dV = \frac{\partial V}{\partial t} dt + \mu S \frac{\partial V}{\partial S} dt + \sigma S \frac{\partial V}{\partial S} dW + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} dt.$$

Here's what each term represents:

- The terms involving dt ($\frac{\partial V}{\partial t} dt$, $\mu S \frac{\partial V}{\partial S} dt$, and $\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} dt$) represent deterministic changes in the option's value over time.
- The term involving dW ($\sigma S \frac{\partial V}{\partial S} dW$) represents the random changes in the option's value caused by the stock price's randomness.

This expanded equation gives the total change in the option value over a small time interval dt, using both deterministic and random components. The Black-Scholes equation is based on the deterministic terms for option pricing. So we need to get rid of randomness

—

Step 3: Constructing a Hedged Portfolio To eliminate risk from randomness (dW), we construct a hedged portfolio:

$$\Pi = V - \Delta S$$

where Δ represents the number of shares held in the portfolio. The goal is to choose Δ such that the portfolio's value Π is no longer affected by random stock price movements.

The change in the portfolio value is:

$$d\Pi = dV - \Delta dS$$

Substituting dV and dS:

$$d\Pi = \left(\frac{\partial V}{\partial t} dt + \mu S \frac{\partial V}{\partial S} dt + \sigma S \frac{\partial V}{\partial S} dW + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} dt\right) - \Delta(\mu S dt + \sigma S dW)$$

Choosing $\Delta = \frac{\partial V}{\partial S}$ cancels the dW term, leaving:

$$d\Pi = \left(\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \mu S \frac{\partial V}{\partial S} - \Delta \mu S\right) dt$$

Substituting $\Delta = \frac{\partial V}{\partial S}$ into the μS term:

$$d\Pi = \left(\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2}\right) dt$$

—

Step 4: Risk-Free Portfolio Under the no-arbitrage assumption, the portfolio Π must grow at the risk-free interest rate r. Thus:

$$d\Pi = r\Pi dt$$

Substituting $\Pi = V - \Delta S$ and $\Delta = \frac{\partial V}{\partial S}$:

$$r(V - S\frac{\partial V}{\partial S})dt = \frac{\partial V}{\partial t}dt + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2}dt$$

Simplifying:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$

This is the Black-Scholes PDE. —

2.3.3 3. Closed-Form Solution for European Call Options

To solve the Black-Scholes PDE, we will impose specific boundary conditions. The solution considers that the option can only be exercised at maturity, T, and that its value depends on the stock price S_0, strike price K, volatility σ, risk-free interest rate r, and time to maturity T.

The closed-form solution for a European call option is derived as:

$$C = S_0 N(d_1) - Ke^{-rT} N(d_2)$$

where:

$$d_1 = \frac{\ln(S_0/K) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}$$

Explanation of Each Term:

- C: The premium of the European call option. This is the value of the option based on the probability-weighted future payoff of exercising the option.
- S_0: The current price of the underlying stock. This term accounts for the value of holding the stock directly.
- $N(d_1)$: The probability (under the risk-neutral measure) that the stock price will exceed a certain level, adjusted for the option's time to maturity and volatility. It accounts for the likelihood of the option finishing in the money.
- Ke^{-rT}: The present value of the strike price. This represents the discounted value of the fixed price the option holder will pay if the option is exercised.
- $N(d_2)$: The probability (under the risk-neutral measure) that the option will be exercised, adjusted for volatility and time to maturity.

Steps to Determine the Closed-Form Solution:

1. **Risk-Neutral Valuation:** In a risk-neutral world, the value of an option is the expected present value of its future payoff. For a call option, the payoff at maturity is:

   $$\max(S_T - K, 0)$$

   Here, S_T is the stock price at maturity or expiration. The value of the option is the discounted expected payoff under the risk-neutral measure:

   $$C = e^{-rT} E[\max(S_T - K, 0)]$$

2. **Lognormal Distribution of Stock Prices:** Under the Black-Scholes assumptions, the stock price S_T follows a log-normal distribution. Using this property, the expectation can be computed analytically.

3. **Breaking the Expectation into Two Terms:** The expected payoff is split into two components:

   $$C = S_0 N(d_1) - Ke^{-rT} N(d_2)$$

   The first term, $S_0 N(d_1)$, represents the expected value of owning the stock if the option is exercised. The second term, $Ke^{-rT} N(d_2)$, represents the discounted value of the fixed strike price to be paid if the option is exercised.

4. **Defining d_1 and d_2:** The terms d_1 and d_2 are derived from the lognormal distribution of stock prices. They represent the standardized z-scores under the normal distribution for the probabilities associated with the option payoff:
$$d_1 = \frac{\ln(S_0/K) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$$
$$d_2 = d_1 - \sigma\sqrt{T}$$
d_1 accounts for the drift and volatility of the stock price over time, while d_2 adjusts for the uncertainty as the option approaches maturity.
5. **Using the Standard Normal CDF:** The cumulative distribution function $N(d)$ calculates the probabilities under the standard normal distribution. These probabilities are weighted by the present values of the stock price and the strike price, yielding the option value.

Interpretation of the Closed-Form Solution: The formula simplifies the complex stochastic process of option pricing into an analytical expression. Each term reflects a specific component of the option's value:

- $S_0N(d_1)$: The expected value of the stock if the option is exercised, adjusted for the likelihood of being in the money.
- $Ke^{-rT}N(d_2)$: The present value of the strike price payment, adjusted for the likelihood of the option being exercised.

The closed-form solution is computationally efficient and widely used in financial markets to price European call options. However, it assumes constant volatility and interest rates, which may limit its accuracy under changing market conditions.

2.3.4 3. Random Numbers: Simulating Noise in Black-Scholes

To reflect real-world uncertainty, we introduce randomness as an extra variable in the Black-Scholes PDE using Gaussian noise. This randomness is applied to simulate fluctuations in parameters, such as interest rates or market conditions, that cannot be accurately predicted. The modified parameter is given by:
$$r_t = r + \eta, \quad \eta \sim N(0, \sigma_{\text{noise}})$$

This approach allows us to account for unpredictable market influences, such as:

- **Fluctuations in interest rates:** Central bank policies or macroeconomic changes that affect borrowing and lending rates.
- **Changes in liquidity:** Variability in market depth caused by shifts in trading volume.
- **Event-driven market disruptions:** Sudden events like earnings announcements, geopolitical tensions, or unexpected news.

By incorporating randomness, the model captures a broader range of possible outcomes, making simulations more representative of real-world market dynamics. This adjustment provides a better framework for analyzing the sensitivity of option prices to changes in external conditions.

—

2.3.5 4. Probability and Statistics: System Performance Analysis

Error Metrics To measure model accuracy, we use:

- **Mean Squared Error (MSE):**
$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}(V_{\text{sim}} - V_{\text{market}})^2$$
- **Root Mean Squared Error (RMSE):**
$$\text{RMSE} = \sqrt{\text{MSE}}$$

Confidence Intervals Confidence intervals provide statistical ranges for the true option premium. For the mean premium:
$$CI = \bar{V} \pm Z\frac{\sigma}{\sqrt{n}}$$
where $\bar{V}$ is the mean premium, Z the Z-score, σ the standard deviation, and n the number of simulations. Narrow intervals indicate reliable results, while wide intervals highlight variability.

Efficiency Metrics We also evaluate:

- **Computation Time:** Time taken to generate results, crucial for large simulations.
- **Model Stability:** Consistency of results under different input parameters.
- **Error Bound Analysis:** Combining error metrics and confidence intervals to assess performance.

These metrics provide a comprehensive picture of the model's accuracy, reliability, and computational efficiency.

2.4 Assumptions

- **Efficient Markets**: Assumes there are no additional costs or taxes affecting trades to simplify the financial modeling process.
- **European Options**: Considers only options that can be exercised at expiration to align with the Black-Scholes framework.
- **Constant Parameters**: Assumes volatility and risk-free rates do not change during the option's life for consistency in calculations.
- **No Dividends**: Excludes dividend payments to focus on pure price movements of the underlying asset.
- **Geometric Brownian Motion**: Models stock prices as following a continuous, random process, which reflects realistic market behaviors.

2.5 Simulation Tools

- **Python Libraries**:
  - **NumPy**: Used for mathematical computations, such as the Black-Scholes formula and random noise generation.
  - **Pandas**: Processes and cleans historical stock data, enabling efficient data manipulation and analysis.
  - **Matplotlib**: Creates clear and customizable visualizations for option prices and trends.
  - **SciPy**: Provides statistical tools, like the normal distribution function, for probability calculations in the Black-Scholes model.
  - **yFinance**: Simplifies downloading historical stock data for input into the model.
- **Polygon.io API**: Fetches accurate, real-time data for the options market, ensuring the model is based on current financial conditions.

The tools were chosen for their precision, compatibility, and ability to handle complex financial data and calculations effectively.

3 Modelling Results

3.1 Technical Details

Implementation of the Model and Running Simulations

The implementation of the model is structured into three main stages: data fetching, option premium calculation, and visualization.

In the **data fetching stage**, stock price data is obtained from Yahoo Finance, and historical option data is retrieved using Polygon.io's API. These datasets serve as inputs for the simulations. Key parameters such as the current stock price, strike price, time to maturity, volatility, and interest rates are defined as constants or derived from the data. The data collected spans the entire year of 2023 for both stock prices and options, providing daily price data for analysis, with a sample size exceeding 250 trading days.

We chose data from 2023 because it represents a relatively stable period following the extreme volatility and market disruptions caused by the COVID-19 pandemic and other global events from 2020–2022. During the earlier years, the stock market experienced unprecedented swings driven by lockdowns, stimulus packages, and shifts in consumer behavior, which introduced significant noise and anomalies into historical trends. By selecting data from 2023, we aim to minimize the influence of these unique events, allowing us to focus on the more typical dynamics of market behavior. This period is more reflective of the current economic and market conditions, making our analysis and simulations more relevant and applicable to present-day scenarios.

In the **option premium calculation stage**, the Black-Scholes formula is used to compute the theoretical value of call options. Randomness is introduced as an extra variable along with variable volatility and interest rates, to simulate real-world market fluctuations. Each simulation involves iterating over the dataset, calculating the option premium for every trading day, and generating a version with randomized noise to better reflect market behavior. This results in hundreds of calculated premiums, including randomized variants, for analysis.

Finally, the **visualization stage** involves plotting the results of the simulations. These include comparisons between real option prices, predicted premiums from the Black-Scholes model, and the randomized premiums. This step helps analyze the accuracy and reliability of the model.

Real Datasets Used

Two datasets are used in the project:

1. **Stock Price Data from Yahoo Finance**: This dataset includes historical daily stock prices (open, high, low, close, and volume) for the SPY ticker. It spans a full calendar year (2023), covering over 250 trading days, with each day containing multiple data points. This data is used to compute the current stock price and as input for the Black-Scholes formula.
2. **Option Data from Polygon.io**: This dataset contains daily aggregate option price data for a specified contract, including the closing prices and timestamps. It also covers the entire year of 2023, providing a similar sample size to the stock price data. The option data is used to compare the simulated premiums against real market option prices.

Rationale for Selecting Recent Data

Using data from 2023 helps ensure that our analysis reflects current market dynamics. The impact of major events like COVID-19, the 2021 stimulus-driven market surge, and the Russia-Ukraine conflict caused abnormal volatility and deviations from historical patterns. These factors created extreme market behavior, which could distort simulations if included. More recent data avoids these outliers and reflects a post-pandemic normalization, providing a clearer picture of typical market conditions.

Additionally, the stock market has been influenced by gradual interest rate hikes and economic tightening policies, making 2023 a more representative year for assessing how options behave under less extreme, but still dynamic, conditions. This ensures our simulations are relevant for present-day market participants and decision-making.

By selecting recent data, we strike a balance between capturing realistic market behavior and avoiding the anomalies introduced by major disruptions. This approach allows us to create simulations and insights that are both accurate and practical for analyzing the effectiveness of the Black-Scholes model under normal economic conditions.

3.2 Results

The Figure shows the graph derived from the actual black scholes open form equation

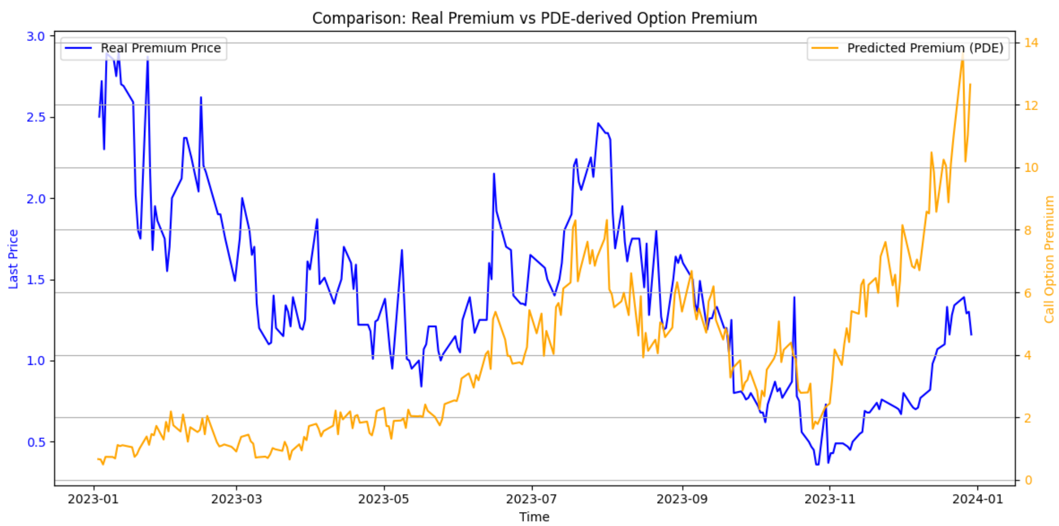

Figure 3: Comparison: Real Premium vs. PDE-derived Option Premium.

As seen above, the PDE-derived option does not perform the best. Below, we introduce the closest form with random guassian noise.

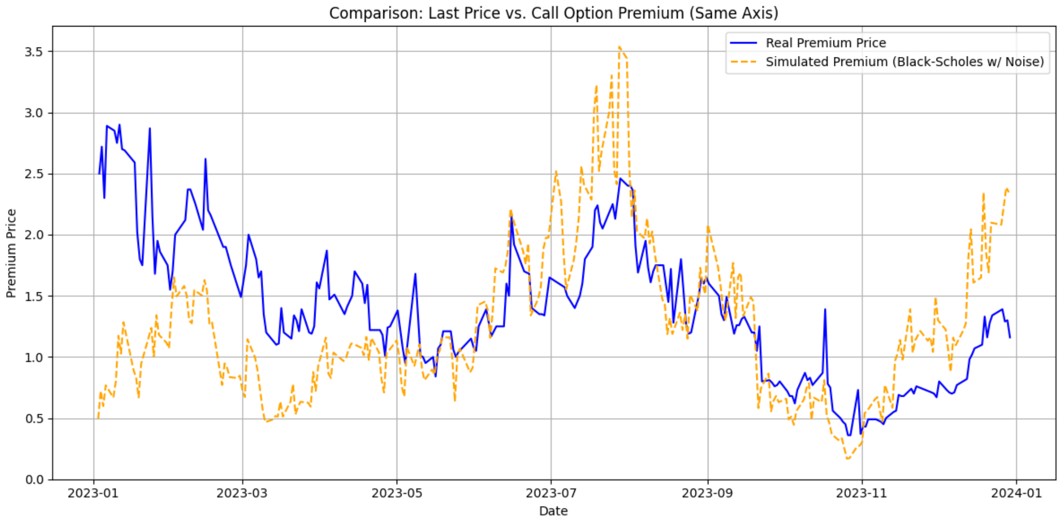

Figure 4: Comparison: Last Price vs. Call Option Premium (with Noise).

The above graph performs significantly better than the open form, finally fixing it's axis and actually having a stable follow across the real premium price. The below graph is the based off the same principal but contains zero random noise.

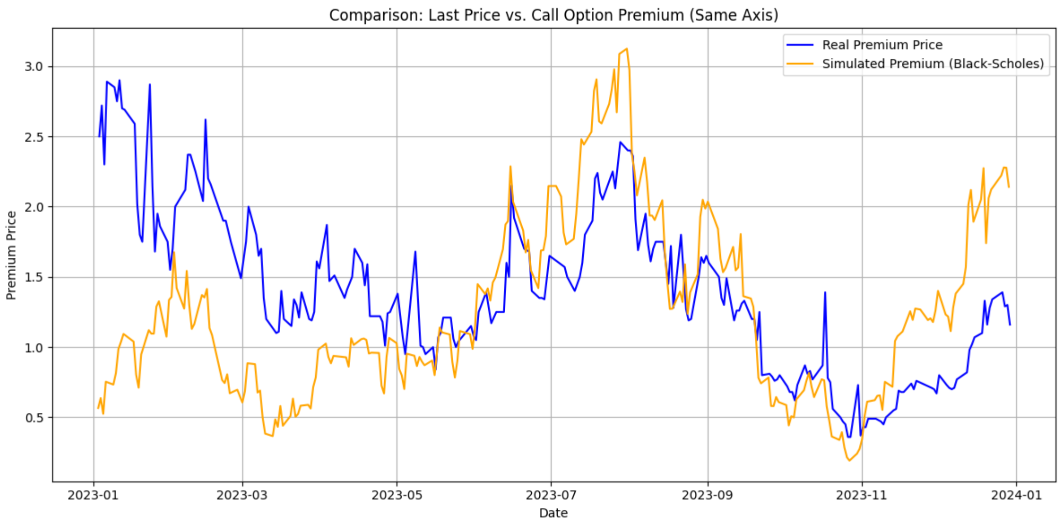

Figure 5: Comparison: Last Price vs. Call Option Premium (with Changing Interest Rate, No Randomness).

The above graph made little to no visual impact but will be analysied later when we calculate the stats. The below graph is again calculated with the closed form of the black scholes equation by now has varying monthly volatility.

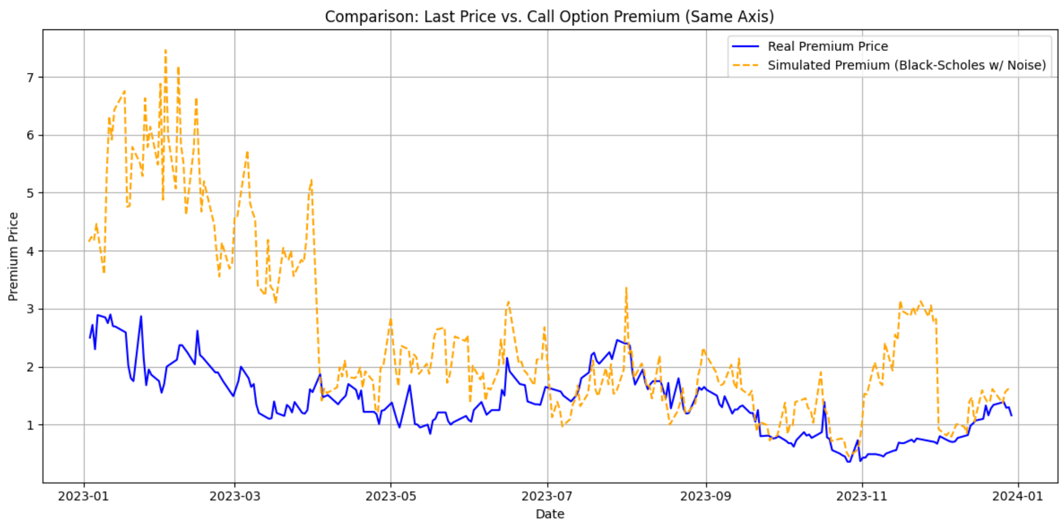

Figure 6: Comparison: Last Price vs. Call Option Premium (Variable Volatility).

THe monthly volatility seems like a overfit and made the model only worse. Below is the same graph but without any random noise.

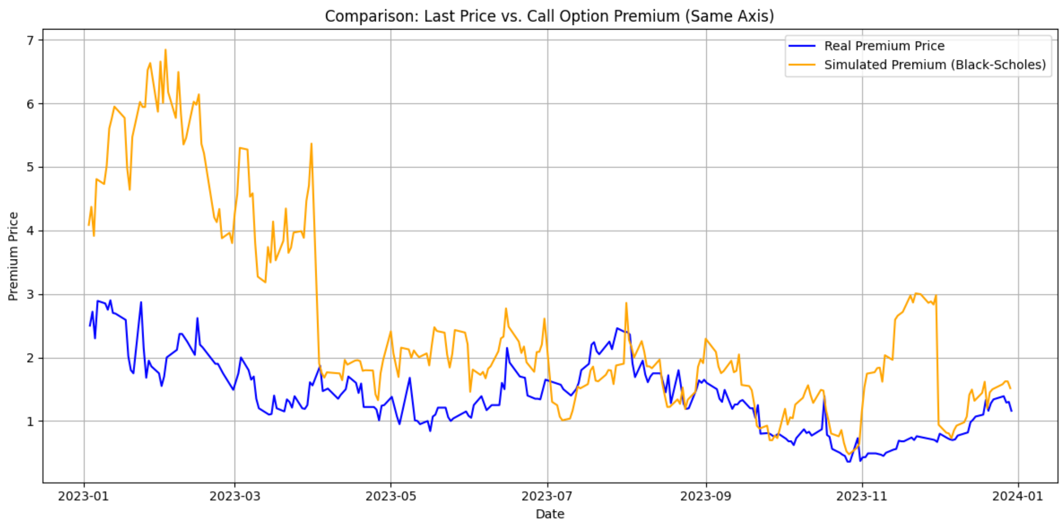

Figure 7: Comparison: Last Price vs. Call Option Premium (with Changing Volatility, No Randomness).

Random noise again like before made little to no visual impact but will be analysied via stats. Below we graph yet again using the closed loop form of the black scholes equation but now varying the interest rates as the US fed changed through the year. This changed happened 4 times in the year.

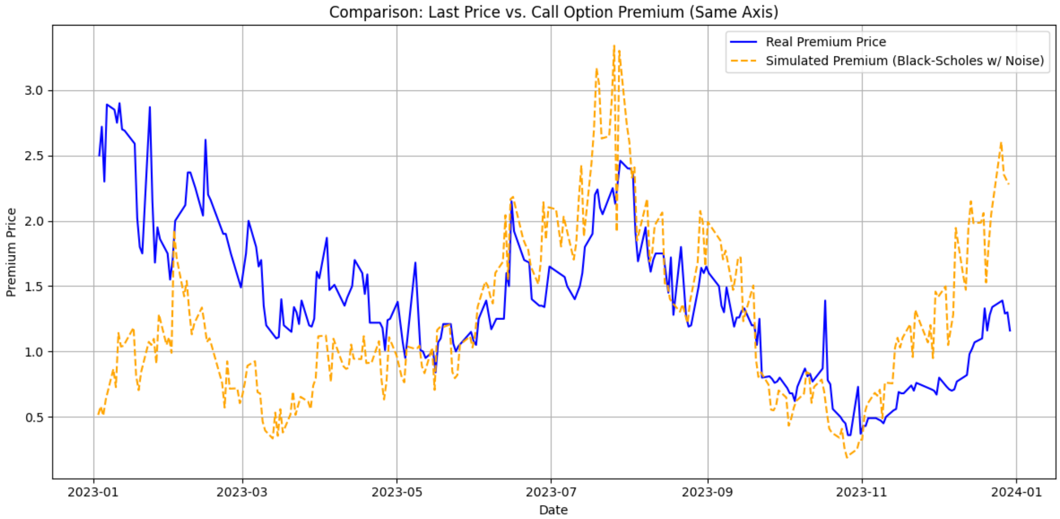

Figure 8: Comparison: Last Price vs. Call Option Premium (Variable Interest Rate).

It appears we have a positive impact, even better than when we weren't varying the interest rates. This will be further analysed via stats.

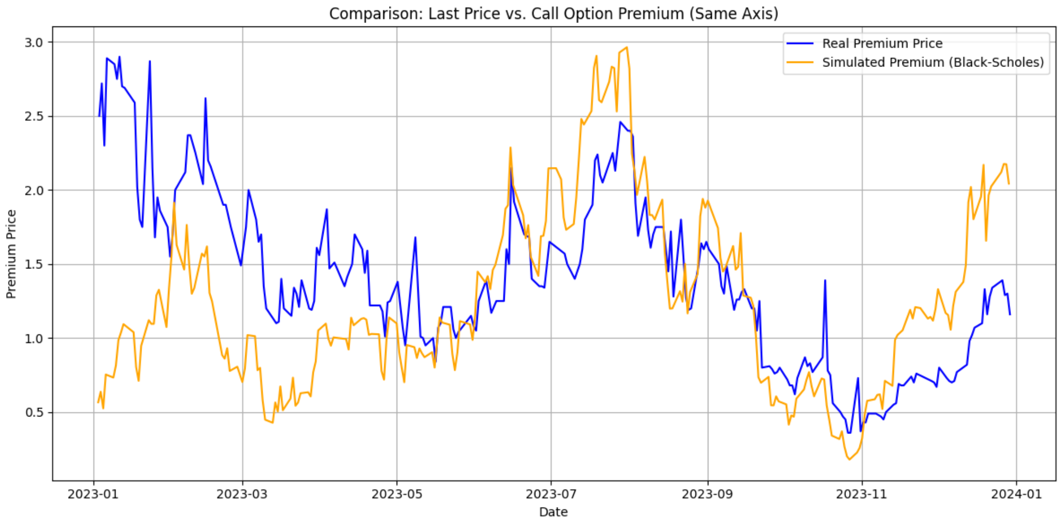

Figure 9: Comparison: Last Price vs. Call Option Premium (Regular Black-Scholes).

Again we have the same model but without any random noise.

Statistics for Original Black-Scholes Predictions

Statistic	Value
Mean Error	0.1476
MSE (Mean Squared Error)	0.3999
RMSE (Root Mean Squared Error)	0.6324
MAE (Mean Absolute Error)	0.4704
MPE (% Mean Percentage Error)	5.3131
Correlation	0.4740
95% CI Mean Error	0.0664 to 0.2287

Table 1: Summary of statistical metrics for the original Black-Scholes predictions.

Statistics for Randomized Black-Scholes Predictions

Statistic	Value
Mean Error	0.1324
MSE (Mean Squared Error)	0.4404
RMSE (Root Mean Squared Error)	0.6636
MAE (Mean Absolute Error)	0.4915
MPE (% Mean Percentage Error)	4.2455
Correlation	0.4505
95% CI Mean Error	0.0466 to 0.2183

Table 2: Statistics for Randomized Black-Scholes Predictions.

Statistics for Original Black-Scholes Predictions with Changing Volatility

Statistic	Value
Mean Error	-1.1102
MSE (Mean Squared Error)	2.9170
RMSE (Root Mean Squared Error)	1.7079
MAE (Mean Absolute Error)	1.1849
MPE (% Mean Percentage Error)	-86.8004
Correlation	0.6064
95% CI Mean Error	-1.2815 to -0.9390

Table 3: Statistics for Original Black-Scholes Predictions with Changing Volatility.

Statistics for Randomized Black-Scholes Predictions with Changing Volatility

Statistic	Value
Mean Error	-1.1155
MSE (Mean Squared Error)	3.0948
RMSE (Root Mean Squared Error)	1.7592
MAE (Mean Absolute Error)	1.2111
MPE (% Mean Percentage Error)	-86.9401
Correlation	0.5965
95% CI Mean Error	-1.2950 to -0.9360

Table 4: Statistics for Randomized Black-Scholes Predictions with Changing Volatility.

Statistics for Original Black-Scholes Predictions with Changing Interest Rate

Statistic	Value
Mean Error	0.1382
MSE (Mean Squared Error)	0.4525
RMSE (Root Mean Squared Error)	0.6727
MAE (Mean Absolute Error)	0.5117
MPE (% Mean Percentage Error)	3.7080
Correlation	0.4249
95% CI Mean Error	0.0514 to 0.2251

Table 5: Statistics for Original Black-Scholes Predictions with Changing Interest Rate.

Statistics for Randomized Black-Scholes Predictions with Changing Interest Rate

Statistic	Value
Mean Error	0.1490
MSE (Mean Squared Error)	0.4805
RMSE (Root Mean Squared Error)	0.6932
MAE (Mean Absolute Error)	0.5216
MPE (% Mean Percentage Error)	3.8791
Correlation	0.3911
95% CI Mean Error	0.0596 to 0.2383

Table 6: Statistics for Randomized Black-Scholes Predictions with Changing Interest Rate.

3.3 Sensitivity to Key Parameters

From the results, it is clear that the model is highly sensitive to changes in volatility. When the volatility varies, we observe a significant increase in errors, such as the Mean Error and Mean Percentage Error (MPE). This suggests that the Black-Scholes model, in its standard form, struggles to handle dynamic market conditions effectively. On the other hand, changing interest rates appear to have a smaller, more contained effect. While they do slightly influence the error metrics, the overall predictions remain relatively stable, highlighting that the model is better equipped to account for interest rate variations than it is for volatility.

Interestingly, the addition of randomization doesn't seem to have a drastic effect on the model's performance. Whether we use the original or randomized Black-Scholes predictions, the metrics are quite similar. This consistency suggests that randomization may not add much value in improving accuracy, at least in this particular application.

3.4 Patterns and Anomalies

One of the most striking patterns is how poorly the model performs under changing volatility. The errors increase sharply, with MPE values becoming highly negative, indicating a systematic underestimation of the target variable. This anomaly highlights a limitation of the Black-Scholes framework—it struggles to adapt when market volatility fluctuates significantly.

In contrast, predictions involving changing interest rates show much more stability. The metrics remain close to those of the static scenario, which implies that the model handles these changes gracefully. This behavior aligns with the theoretical underpinnings of the Black-Scholes model, which incorporates interest rates more naturally into its formulation.

Another noteworthy observation is the correlation between the model's predictions and actual outcomes. In scenarios with changing volatility, the correlation improves slightly, suggesting that while the errors are large, the model may still capture some underlying trends. However, when randomization is introduced, the correlation tends to drop slightly, indicating that the added randomness may introduce noise rather than meaningful improvements.

3.5 Initial Insights

Overall, the results reveal a clear challenge: the Black-Scholes model excels in static, controlled conditions but struggles when faced with dynamic environments, particularly with fluctuating volatility. This limitation is important to consider when applying the model to real-world financial data. On the other hand, the model's relative stability under changing interest rates is reassuring and shows that it can handle certain market dynamics well.

These observations suggest that while the Black-Scholes model remains a valuable tool, its performance can vary significantly depending on the specific market conditions. This highlights the importance of understanding the limitations of any financial model and considering enhancements or alternatives when necessary.

4 Appendix: Program Code

`https://github.com/rehanthestar21/m-s_final_coursework`

5 Discussion and Conclusion

Our goal in this project was to evaluate different versions of the Black-Scholes model to determine their effectiveness in predicting real-world option prices. Along the way, we identified key insights into what approaches worked well and what did not.

5.1 Discussion

Open-Loop PDE Black-Scholes Model: The open-loop PDE version of the Black-Scholes model did not perform as well as the other configurations. While its predictions maintained a general shape similar to actual premium prices, the axes were misaligned, and its overall accuracy was poor. Given its limited promise, this approach was not further analyzed in detail.

Closed-Form Black-Scholes Model: The closed-form Black-Scholes model provided a reliable baseline for comparison. By introducing randomness (or "noise"), the model's predictions improved significantly. This adjustment allowed it to better capture the unpredictable fluctuations seen in real-world markets. The findings highlight the importance of incorporating randomness to reflect market dynamics more accurately.

Changing Volatility: Adjusting volatility every 30 days did not yield favorable results, particularly for short-term predictions. The model struggled to align with real prices over shorter intervals, suggesting that market prices are less sensitive to short-term volatility changes. This indicates the need to consider volatility over longer time horizons to improve predictive accuracy.

Impact of Interest Rates: One of the most impactful improvements came from incorporating changes in interest rates. This adjustment significantly enhanced the model's alignment with real market data, demonstrating the critical role of interest rate variability in financial modeling.

5.2 Conclusion

This project underscores the importance of adapting financial models to reflect the complexities of real-world markets. While the open-loop PDE approach proved less effective, the closed-form Black-Scholes model demonstrated its utility, especially when enhanced with randomness and interest rate adjustments.

Key takeaways include:

- **Role of Randomness:** Adding noise improved the model's performance by accounting for the unpredictability of market conditions.
- **Volatility's Long-Term Impact:** Short-term volatility adjustments were less effective, highlighting the need for a long-term perspective.
- **Significance of Interest Rates:** Accounting for interest rate changes greatly improved the model's accuracy, emphasizing their importance in simulations.

Future work could focus on developing more advanced techniques to model stochastic volatility and dynamic interest rates. Additionally, leveraging machine learning or hybrid approaches may further enhance the model's predictive capabilities and adaptability to real-world market conditions.

References

[1] Hull, J. C., *Options, Futures, and Other Derivatives*, Pearson, 2017.
This is one of the most widely used textbooks in finance and covers the Black-Scholes model, option pricing, and the dynamics of financial markets.

[2] Black, F., & Scholes, M., ”The Pricing of Options and Corporate Liabilities”, *Journal of Political Economy*, 1973.
The foundational paper that introduced the Black-Scholes model.

[3] **Investopedia**, `https://www.investopedia.com`.
A comprehensive source for financial concepts, including stock options, volatility, and option pricing models.

[4] **CBOE Global Markets**, `https://www.cboe.com`.
The Chicago Board Options Exchange provides information on how options are traded and the factors influencing option prices.

[5] Mandelbrot, B., ”The Variation of Certain Speculative Prices”, *The Journal of Business*, 1963.
Explores the randomness and patterns in market prices, which influence volatility modeling.

[6] Heston, S. L., ”A Closed-Form Solution for Options with Stochastic Volatility”, *The Review of Financial Studies*, 1993.
Discusses models that extend Black-Scholes by incorporating randomness in volatility.

[7] **National Bureau of Economic Research**, `https://www.nber.org`.
Features studies on market dynamics, volatility, and macroeconomic impacts on financial markets.

[8] **Bloomberg Terminal**.
A professional tool used to analyze option pricing, volatility, and market data.

[9] **Yahoo Finance**, `https://finance.yahoo.com`.
Provides historical stock and option price data used for modeling.

[10] **Polygon.io**, `https://polygon.io`.
Offers real-time and historical market data.